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ADVANCES OF DAMPING ESTIMATION FOR ENGINEERING STRUCTURES –I

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ABSTRACT

Damping plays a key role in dynamic response prediction of the structures subjected to broadband excitation, and vibration control based on energy dissipation. Two major dynamic testing approaches have been surveyed. A variety of damping estimation methods via Logarithmic Decrement (LogDec), Hilbert Transform (HT), Wavelet Transform (WT), Data correlation and Enhanced Frequency Domain Decomposition (EFDD), etc. have been overviewed. Their performances in dealing with noisy data and multi-modes are studied in principal and via numerical simulation. Major issues in damping estimation are also discussed.

1. Introduction:

Damping is of great importance in structural dynamics, especially in dynamic response prediction and vibration control. Dynamic response of a structure is determined by dynamics characteristics of the structure and external loads. Resonance plays a key role in the dynamic response especially for lightly damped structural system. Two critical parameters of resonance are resonance frequency and damping ratio, which are determined by mass/stiffness and damping characteristics of the system. In aerospace and civil engineering, structures are often induced by broadband excitation, such as turbulence, wind gust, traffic, tremor, etc. Therefore, damping is the major reason causing excessive vibration or large dynamic response in the structural system.

Structural vibration control becomes increasingly important for aerospace and civil engineering structures. No matter different ways of implementation (passive, active or semi-active), the major two mechanisms of vibration control are energy dissipation and vibration isolation. The effects of energy dissipation enhancement may be achieved either by conversion kinetic energy to heat (damping), or transferring of energy to other device (dynamic absorbing). A variety of innovative devices for supplement damping, such as dampers based on friction, material yield, viscoelastisity and viscous fluid, tuned mass and tuned liquid damper, as well as active control systems have been developed in recent years. Effectiveness of these devices should be evaluated and verified by damping enhanced.

A dynamic structure can be damped by mechanisms with different internal and external nature: friction between atomic/molecular or different parts, impacts and air/fluid resistance, etc. A combination of different phenomena results in various types of damping. Generally, their mathematical description is quite complicated, and not suitable for vibration analysis for complicated structures. In structural dynamics, damping is described via viscous, hysteretic, coulomb and velocity squared model. Viscous damping occurs when the damping force is proportional to the velocity. Hysteretic damping, also known as structural damping, is associated with the hysteresis loop. It is frequency dependent and acts with a force that is proportional to displacement. Coulomb damping force caused by the friction opposes the direction of motion. Velocity squared damping occurs when a vibrating object is subjected to air resistance.

Coulomb and velocity-squared damping models are nonlinear. Hysteretic damping is limited to steady state vibrations. Viscous damping is mathematically convenient because it results a linear second order differential equation for engineering structures. A transient decay of a viscously under damped system will decay exponentially. From practical point of view, **equivalent viscous damping**, which models the overall damped behavior of the structural systems as being viscous, is often adopted in structural dynamics [1]. Even though, analytical modeling of damping is still very difficult, if not impossible, for real world structures. Therefore experimental **damping estimation** becomes extremely important in structural dynamics, especially for the purpose of dynamic response prediction and vibration control.

2. Dynamic Testing & Data Pre-processing

2.1 Dynamic Testing

Dynamic characteristics, including damping, of an engineering structure can be obtained experimentally via dynamic testing. Traditional dynamic testing is conducted in the laboratory environment, where artificially controlled excitation force is applied and responses are measured. There are variety of excitation forces can be used in dynamic testing, e.g. steady-state harmonic sweep excitation, transient (step or impulse) and random excitations. The former is featured single frequency excitation. The latter two have broadband spectrum and can excite all modes in the frequency range of interest simultaneously.

Field, or on-site, test is often required in aerospace and civil engineering. Artificial excitation can be applied in field-testing, e.g. control surface or special winglet excitation in aircraft flight flutter testing; mechanical, electric or hydraulic shaker excitation for buildings and bridges, etc. When artificial excitation is applied, the dynamic responses and excitation force can be measured as in the lab testing. In some cases, e.g. transient excitation, only output data can be obtained.

In many occasions, ambient, or natural, excitation is preferred in field-testing. Actually, ambient testing has plenty of advantages compared to lab testing. Ambient testing is cheap and fast, no elaborate excitation equipment required, no boundary condition simulation needed; Dynamic characteristics of the whole system, instead of component, can be obtained directly; During ambient testing the structure is under operational condition and is subjected to real external loading, which is usually differ significantly from the excitation in lab testing; Ambient testing can be utilized not only for dynamics characterization of the structure, but also *in-situ* vibration-based health monitoring and active vibration control.

2.2 Data Pre-processing

In the dynamic testing, where input/output data can be measured, Frequency Response Functions (FRFs) are usually estimated by Fast Fourier Transform (FFT), or transformed in time domain (TD) to obtain Impulse Response Function (IRF) by inverse FFT. Quite a few FRF estimation techniques have been developed in the last two decades, such as H_1, H_2 estimation, vector FRF estimation H_v and Total Least Squares FRF estimation H_T [2], among others. H_1 estimator is based on assumption that noise is only in output signal, and will have bias error when input signal is polluted by noise. H_2 estimator is vise versa. H_v and H_T are supposed to be unbiased when both input and output signals contain noises.

In the output only cases, data pre-processing is normally utilized. In ambient test, broadband responses are measured. Auto and Cross Correlation Function (COR) can be estimated via direct FFT computation. It should be noticed that COR estimation via FFT computation is much faster compare to direct computation, but will usually results bias error. Zero-padding technique can be used to obtained unbiased estimates [3]. It is noticed that the CORs are also an exponentially decaying sinusoid, as IRF*s*.

Random Decrement (RDD) technique is another effective way to estimate exponentially decayed TD response function of the system. RDD signature can be calculated by average of triggered random response data. The basic concept is that a random response of a linear structure consists of deterministic part (e.g. free decay) and a random part, which approach to zero after averaging and leave free decay response [4]. Theoretical explanation of RDD is developed and shows that RDD is actually a correlation function [5]. A diversity of techniques to calculate RDD has been studied [6]. In the theoretical development, it is assumed that the inputs to the structure are zero-mean stationary white noise process. But it is shown this strict condition is not always necessary.

In the dynamic testing with transient excitation, the Free Decay Response (FDR) can be directly measured for modal parameter estimation. In this paper, Time Response Function (TRF) is defined in wide meaning, and consists of IRF, FDR, RDD signature, as well as COR.

In summary, the dynamic testing can be classified into two groups: the testing with input/output measurements, and output only. The major objective of data pre-processing is to obtain TRFs for further modal parameter (including damping ratio) estimation:

- FRF or IRF estimated from both input/output measurement;
- COR and RDD signature estimated from random response;
- FDR and impulse response directly measured from transient response;

3. Damping Estimation Methods

3.1 Half Power method

In the dynamic test, where both input/output can be measured, FRF can be estimated via FFT. The simplest technique to estimate damping ration is Half-Power method. Half power bandwidth is the difference of the two frequencies on either side of a resonance frequency, where the amplitudes equal 0.707 (-3dB) of the peak amplitude. The damping ratio can be determined from formula: $\zeta = \Delta f_{dB} / 2 f_r$. Actually, the half power bandwidth, and damping ratio, can be more accurately obtained from imaginary or real part of FRF, or from Nyquist plot in complex plane.

The Half Power method can also be applied in the output only cases, where only Power Spectrum Density (PSD) can be measured infrequency domain.

3.2 Logarithmic Decrement (LogDec) method

The logarithmic decrement (LogDec) method is commonly used to give a quick estimate of the damping ratio from a Time Response Function (TRF) of S-DOF system, represented by an exponentially decayed sinusoid (Figure 1),

$$x(t) = A_r e^{-\sigma_r t} \cos(\omega_{dr} t + \varphi_0) \tag{1}$$

Where σ_r, ω_{rd} are decay rate and damped natural frequency of the r-th mode, and $\zeta_r = \omega_r / \sigma_r$, $\omega_r = \sqrt{\omega_{dr}^2 + \sigma_r^2}$ are damping ration and modal frequency, respectively.

LogDec is defined as the natural logarithm of the ratio of any two successive decay amplitudes,

$$\delta_r = \ln \frac{X_i}{X_{i+1}} = \frac{2\pi\zeta_r}{\sqrt{1-\zeta_r^2}}, \quad \zeta_r \cong \frac{\delta_r}{2\pi} \tag{2}$$

When the decay signal is polluted by noise, more amplitude points are chosen and plotted in semi-logarithmic plane; a straight line is resulted by linear regression (Figure 2). The decay rate can then be obtained as its slope, and the damping ratio is calculated from decay rate divided by the modal frequency.

When dealing with a multiple degree of freedom (M-DOF) system, one can filter out the effect of other modes in the frequency domain (FD) and transform the signal back into the time domain (TD). The accuracy of the method breaks down when there are two close modes.

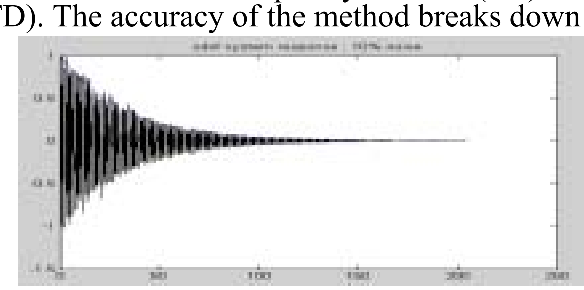

Fig. 1 Exponentially decayed TRF

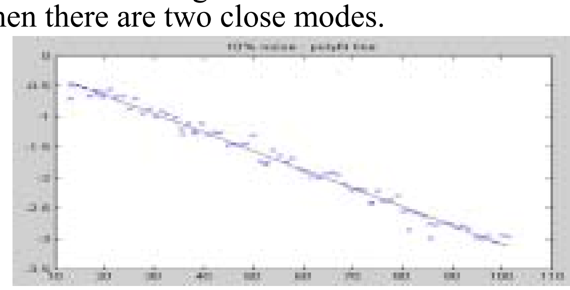

Fig. 2 Peaks from noise polluted data with linear regression in LodDec

3.3 Hilbert Transform (HT) method [7],[8]

Hilbert Transform (HT) is defined as following convolution

$$x^H(t) = H\{x(t)\} = x(t) * \frac{1}{\pi t} = \frac{1}{\pi}\int_{-\infty}^{\infty} x(t)\frac{1}{t-\tau}d\tau \quad (3)$$

An analytic signal of $x_a(t)$ is defied as

$$x_a(t) = x(t) + jx^H(t) \quad (4)$$

A complex signal $x_c\ (t)$ is assumed to be an approximation of the analytic one,

$$x_a(t) \cong x_c(t) = A(t)e^{j\varphi(t)} \quad (5)$$

Where the modulus $A(t)$ represents the instantaneous envelope, while the argument $\varphi(t)$ is the instantaneous phase and given by

$$A(t) = \left((x(t))^2 + \left(x^H(t)\right)^2\right)^{1/2}, \quad \varphi(t) = \tan^{-1}\left(x^H(t)/x(t)\right) \quad (6)$$

When $e^{-\zeta_r\omega_r t}$ is a slowly varying function, the envelop in the semi-log plane is a straight line,

$$\ln(A_r e^{-\zeta_r \omega t}) = -\zeta_r \omega_r b + \ln(A_r) \quad (7)$$

The slope of which is the decay rate (Figure3). In the similar way, the damped natural frequency can be obtained as the slope of the instantaneous phase line (Figure 4).

In the HT method, only one data block with sufficient points is required in order to obtain a good estimate of damping ratio, even noise is presented. Linear regression averages out zero-mean stationary noise. Numerical simulation shows clearly that the accuracy of damping ratio estimation by HT method is much higher than LogDec.

However, it should be noted that HT is an approximate method because the analytic assumption. It is accurate only when the system is conservative. Furthermore, it is important to emphasize that, HT, as LogDec, works theoretically only for single DOF system. In practice, HT method can be employed for system with medium coupled modes when band pass filter is adopted.

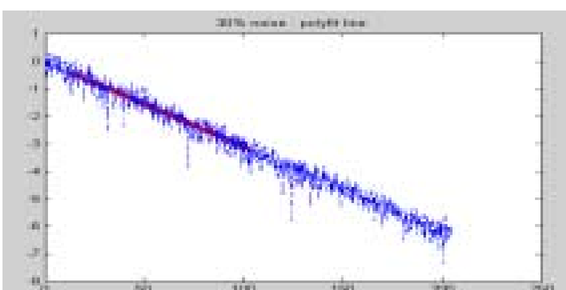

Fig. 3 Linear regression to fit noisy decay

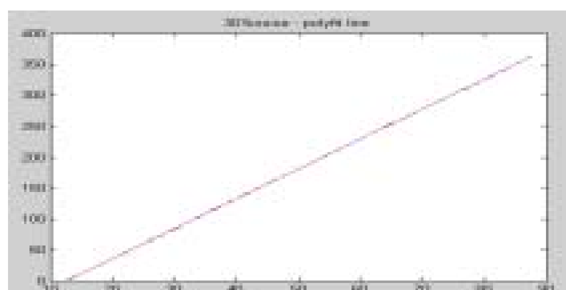

Fig. 4 damped natural frequency from phase

3.4 Wavelet Transform (WT) method

Wavelet Transform (WT) of a signal is a time-scale decomposition obtained by dilating and translating a selected basic wavelet function. The continuous wavelet transform (CWT) is defined as follows,

$$w_x(a,b) = \frac{1}{\sqrt{a}}\int_{-\infty}^{\infty} x(t)g^*\left(\frac{t-b}{a}\right)dt \quad (8)$$

Where b is the translation parameter indicating the locality in time domain, a is a dilation or scale parameter localizing the signal in frequency domain, $g(t)$ is an analyzing function called basic wavelet, $g^*(t)$ is its complex conjugate. The possibility of time-frequency localization arises from the basic wavelet being a window function, which decays very fast like a "small wave",

$$\int_{-\infty}^{\infty} g(t)dt = 0 \quad (9)$$

WT of a signal actually is a convolution of the signal with the selected wavelet function, and can be calculated in frequency domain via FFT.

$$w_x(a,b) = \sqrt{a}\int_{-\infty}^{\infty} X(f)G_{a,b}^{*}(af)e^{2\pi fb}df \quad (10)$$

One of the most widely used wavelet functions for structural parameter identification is the well-known Morlet wavelet [8]

$$g(t) = e^{-t^2/2}e^{j\omega_0 t} \quad (11)$$

Its spectrum shows excellent frequency locality (Figure 5). For an exponentially decayed time response function (TRF) $x(t)$, its WT with Morlet wavelet can be derived as

$$w_x(a,b) = \sqrt{2\pi a}Ae^{-\sigma_r t}e^{-2\pi^2(af-f_0)^2}e^{j2\pi f_{dr}b} \quad (12)$$

When choosing dilation parameter as $a=f_0/f$ and logarithm is applied, the modulus of the wavelet transform can be written as

$$\ln(|w_x(a,b)|) = -\zeta_r\omega_r b + \ln(AG_{a,b}^{*}(ja\omega_{dr})) \quad (13)$$

Thus the damping ratio can be estimated from the slope of the straight line of the wavelet modulus cross-section for given value of dilation a, plotted in a semi-logarithmic plane. The rest procedure of damping ration estimation is similar to the envelope analysis used in HT method.

The above formulation is derived from complex-valued analytic signal. However, it can be extended easily to the exponentially decayed TRF. Numerical simulation of WT method has shown very encourage results, even with very noisy TRF data (Figure 6). Moreover, WT posses two plausible features: one is that it works for multiple DOF systems; the other is the applicability with non-stationary measurement.

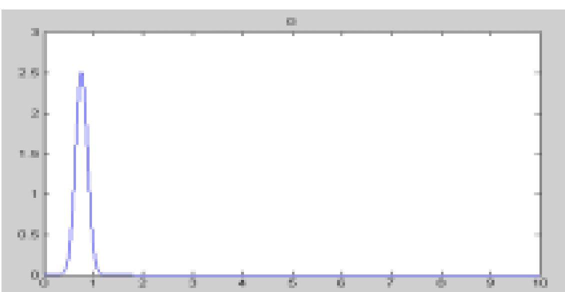

Fig. 5 The spectrum of Morlet wavelet shows excellent frequency locality

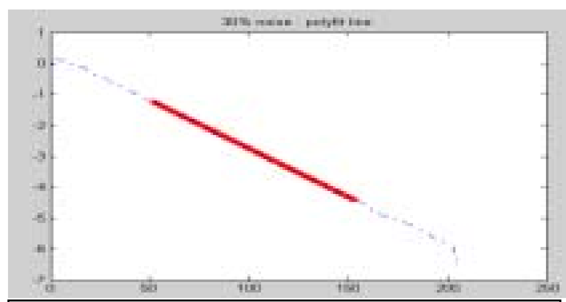

Fig. 6 Robust decay rate estimation obtained from wavelet modulus cross-section,

Two other WT-based techniques have been developed recently [9]. One procedure can recover single mode time response function (as exponential decayed sinusoid) via WT; damping is then estimated as a slop of the semi-logarithmic plot as in LogDec technique. The other one detects the ridge of the WT for each mode. Thus the skeleton of the WT gives its TRF and its Hilbert transform. This can be used to obtain instantaneous envelope function to estimate the damping ratio for each mode.

3.5 Damping Estimation with Data Correlation

It has been shown [10] that data correlation, or correlation filtering, is very powerful in noise reduction. When enough TRF data points are available, their correlation function can be computed and used as new TRFs for further parameter estimation. The zero-mean uncorrelated noise in the signal will then be “filtered” out very effectively. Accurate results were obtained by

data correlation when using the same numerical simulation examples even with very noisy TRF data (Figure 7, 8).

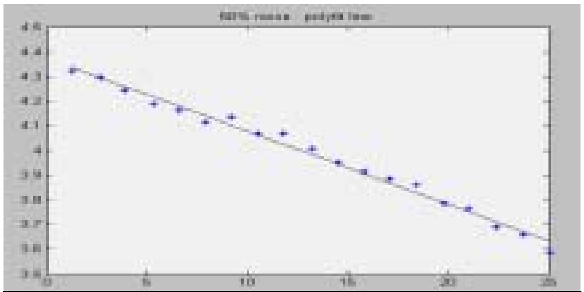

Fig. 7 Damping estimation via data correlation of LogDec, 50% noise. See effectiveness

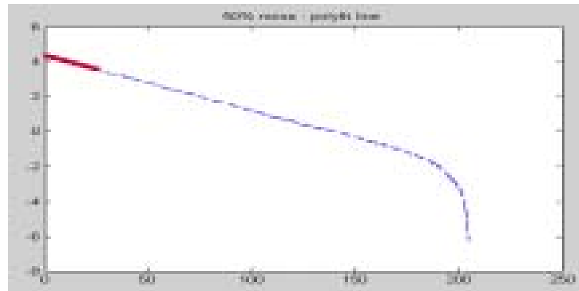

Fig. 8 Damping estimation via data correlation of HT, 50% noise. See Noise reduction

3.6 Enhanced Frequency Domain Decomposition (EFDD) method

Heavy modal coupling is often encountered when testing a complex engineering structure. Symmetrical and anti-symmetrical modes of a structure are often closely spaced. Axially symmetrical structure even posses modes with almost identical modal frequencies. Large complicated structures, e.g. spacecrafts and long-span flexible bridges, have dense modes in the frequency range of interest. All above damping estimation methods will have difficulty in dealing with closely spaced modes. A new method called Enhanced Frequency Domain Decomposition (EFDD) has been developed to resolve the difficulties [11],[12].

EFDD is also a 2-stage parameter estimation approach. In the first stage auto and cross PSDs are estimated from all response measurements. Singular value decomposition (SVD) of the PSD matrix is then conducted to obtain a set of singular value (SV) spectrum (Figure 9). It is proved that the data in the vicinity of the peaks of the first SV spectrums represent the enhanced PSD of the system. It is also revealed that the PSD matrix is equivalent to FRF matrix when the system is subjected to broadband excitation. The inverse FFT of the enhanced PSD is a very close approximation of TRF! (Figure 10) Therefore, the previously introduced damping estimation methods can be adopted in the second stage.

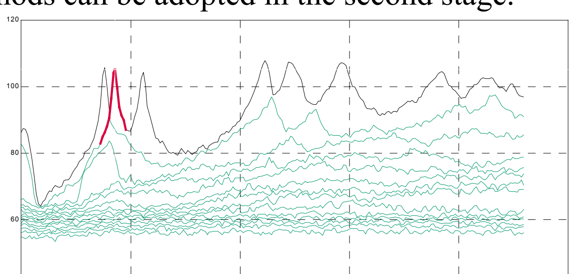

Fig. 9 SV Plot from a tall building. "Bell" shows the enhanced PSD data of 2-nd mode

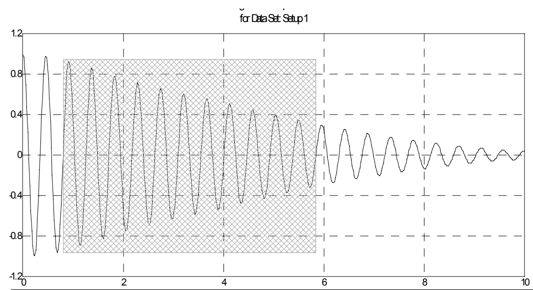

Fig. 10 Time response function, very close to the IRF of the second mode

In essence, EFDD is a spatial domain method. It can handle not only closely spaced modes, but also modes with repeated modal frequencies. Another significant advantage of this technique is that EFDD is very robust to the noise pollution, because the uncorrelated noise is dramatically reduced during the operation of SVD on polluted PSD data.

4. Numerical Simulations

A set of exponentially decayed response data was created to simulate the TRFs of the first bending mode (f_1=0.74 Hz) of a 15-story office building [12] are utilized for numerical simulation to show the performance of the different damping estimation methods described above. Table 1 summarized the results of LogDec, HT and their data correlation (R-LogDec, R-HT), as well as WT methods using noise polluted TRF data. Table 2 gives the numerical simulation results of a communication transmission tower, which has three modes with modal frequencies of 1.20, 2.65 and 3.85 Hz. The WT method was employed for damping estimation of the M-DOF system.

Table 1 Numerical Simulation Results of Damping Estimation for the first mode of a tall building

		Target	Damping Estimation Results with Noisy TRF data					
			5%	10%	30%	50%	70%	100%
X-LOG	Damp (%)	0.6500	0.6469	0.6098	0.4244	0.3287	0.2804	0.2414
	Error. (%)	/	-0.48	-6.2	-34	-49	-57	-63
X-HT	Damp (%)	0.6500	0.6501	0.6518	0.6511	0.5921	0.4862	0.4003
	Error. (%)	/	0.015	0.28	0.17	-8.9	-25	-38
R-LOG	Damp (%)	0.6500	0.6473	0.6470	0.6525	0.6490	0.6450	0.6719
	Error. (%)	/	-0.18	-0.46	0.38	-0.15	-0.77	3.4
R-HT	Damp (%)	0.6500	0.6496	0.6534	0.6529	0.6355	0.6589	0.6307
	Error. (%)	/	-0.061	0.52	0.45	-2.2	1.4	-3.0
X-WT	Damp (%)	0.6500	0.6500	0.6500	0.6592	0.6463	0.6545	0.6656
	Error. (%)	/	0.0	0.0	1.4	-0.57	0.69	2.4

Table 2 Numerical Simulation Results of Damping Estimation for a Transmission Tower

		Target	Damping Estimation Results with Noisy TRF data					
			5%	10%	30%	50%	70%	100%
1st Mode	Damp (%)	1.0000	0.9531	0.9576	0.9715	0.9612	0.9632	0.9519
	Error. (%)	/	-4.7	-4.2	-2.9	-3.9	-3.7	-4.8
2nd Mode	Damp (%)	0.8000	0.7832	0.7797	0.7644	0.7260	0.7525	0.6794
	Error. (%)	/	-2.1	-2.5	-4.4	-9.3	-5.9	-15
3rd Mode	Damp (%)	0.5000	0.4886	0.5002	0.4866	0.4407	0.4716	0.4622
	Error. (%)	/	-2.3	0.048	-2.7	-12	-5.7	-7.6

6. Concluding Remarks

- We have classified dynamic testing into two categories. In the input/output cases, FRFs or IRFs are normally estimated via FFT technique for sweep sine, transient or random controlled excitation. There are two output only cases: random and transient excitation. When the structures are subjected to ambient excitation with broadband spectrum, correlation (covariance) functions (COR) are estimated from random response for further parameter estimation. In the case of transient excitation without input force measurement, the free decay response can directly be adopted for further parameter estimation.
- A variety of damping estimation methods via Logarithmic decrement (LogDec), Hilbert Transform (HT), Wavelet Transform (WT), Data correlation and Enhanced Frequency Domain Decomposition (EFDD), etc. have been overviewed in this paper. Basically they are 2-stage approach. Estimation of FRF or IRF, COR, or direct make use of FDR as the first stage. IRF, COR and FDR are defied as Time Response Function (TRF) in this paper and utilized for damping estimation as the second stage.
- Numerical simulations of the first bending mode of a tall building and multi-modes of a communication transmission tower were conducted to show and compare performances of different damping estimation methods. LogDec is the simplest one and performs fairly well with light or medium noise contamination, e.g. less than 10%, in TRF data. HT method can get satisfactory estimates when noise increased to 50 %. WT method can achieve most accurate results with noisy data. Data correlation shows great advantage in noise reduction of TRFs, which is extremely effective for accuracy of damping estimation. The limitation is more data is required for data correlation. WT method has also successfully applied to real data measured from ambient test of a tall building and transmission tower.
- Theoretically, LogDec and HT methods work only for S-DOF system. Making use of a suitable band pass filter, pretty accurate results can be obtained for lightly or medium coupled modes. One of the attractive features of WT method is that it can deal with multi-

modes directly owing to its time-scale decomposition. EFDD is a simple but powerful method in modal parameter estimation, which has the ability to deal with closely spaced modes, even the modes with almost identical modal frequencies.

- All the damping estimation techniques overviewed in this paper can be classified as non-parametric method. An array of model-based parametric modal parameter estimation procedures has been developed the last 30 years. The parametric modal identification works for both input/output and output only measurement, and can handle closely spaced modes. Satisfactory modal frequency and mode shapes can be achieved. However, there are lots of challenges for accurate damping estimation. The advances of parametric damping estimation will be the content of the other sister paper to be published latter.
- The accuracy of the damping estimation for engineering structures is an important issue for further study. There are many error sources, bias and variance, in TRF estimation. For example, whenever FFT is adopted to calculate TRF, leakage will occur and causes significant bias error in the next damping estimation stage. Variance error in TRF, which increases when data length is limited, will also result in bias error in damping estimation. Meanwhile, damping estimation it-self may produce bias and/or variance error too.
- Another important issue in damping estimation is in theoretical aspect. When the damping is proportional, normal mode is result, i.e. damping matrix can be diagonized by modal matrix of undamped system. And therefore M-DOF system can be decomposed into S-DOF system in modal space. However, in real engineering structures, damping is hardly to be proportional. Then the modal frequency obtained from complex modal analysis is NOT the undamped natural frequency! Therefore, we need either an appropriated measure for non-proportional damping, or even new description of the damping property.

REFERENCES

[1] TOMPSON, W.T. *Theory of Vibration with application*, 3rd Ed. Prentice Hall, Englewood Cliff, New Jersey, 1993

[2] Zhang, L.-M. "Assessment of Frequency Response Function Estimation & Total Least Squares FRF Estimator", *Chinese Journal of signal Processing*, 5(3), 1989

[3] BENDAT, J. & PIERSOL, A., *Random Data, Analysis and Measurement Procedures*, John 'Wiley & Son, New York, 1986

[4] IBRAHIM, S. R. "Random Decrement Technique for Modal Identification of Structures," *Journal of Spacecraft and Rockets*, Vol. 14, 1977, pp. 696-700

[5] VANDIVER, J. K. et al., A Mathematical Basis for Random Decrement Vibration Signature Analysis Technique, *ASME J. of Mechanical Design*, Vo. 104, April 1982

[6] ASMUSSEN, J.C., Modal Analysis based on the Random Decrement Technique, Ph.D. Thesis, Aalborg University, Denmark, 1997

[7] AGNENI, A. and BALIS CREMA, L. Analytic Signal in the Damping Coefficient Estimation, *Proc. Of the Int'l Conf. On Spacecraft and Mechanical Testing*, Noordwijik, The Netherlands, 1988

[8] RUZZENE,M et. al. Natural Frequency and Damping Identification using Wavelet Transform, *Mechanical System& Signal Processing,* 11(2), 1997

[9] LAMARQUE,C.H. et. al. Damping Identification in M-DOF system via Wavelet-Logarithmic Decrement-I: Theory, *J. of Sound and Vibrat*ion, 235(3), 2000

[10] ZHANG, L.-M, YAO, Y.-X. An Improved Time Domain Polyreference Modal Identification Method, *Mechanical System & Signal Processing*, 1 (3), 1987

[11] BRINCKE, R., ZHANG, L.-M. and ANDERSON, P., Modal Identification from Ambient Response using Frequency Domain Decomposition, *Proc. of the 18th IMAC,* San Antonio, TX, USA, Feb. 7-10, 2000

[12] TAMURAR,Y., ZHANG,L.-M, et. Al. A. Ambient Test & Modal Identification of an Office Building, *Proc. of the 20th IMAC*, Los Angeles, USA, Feb. 2002