Effects of Fluctuating Wind Loading on the Stability of Single-Layer Reticulated Shells

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ABSTRACT: Wind loading is very important to single-layer reticulated shells since this structural system is sensitive to external loading distribution and stability is a dominant problem for their structural design. Usually an equivalent static method is used for wind-resistant design. However, the estimated equivalent static wind loading may not reflect the actual effect of fluctuating component on the stability of shells. In this paper, based on wind pressure data measured simultaneously in wind tunnel, the effects of fluctuating wind loading on structural limit load-carrying capacity and stability of shells were investigated. A framework to estimate the effective wind loading distribution was introduced, and a new method from the stability point of view was presented to improve the efficiency of the effective static loading distribution estimation, as well as to give a conservatively estimation on the effect of wind loading. With comparative analyses, the advantage of the presented method was shown finally.

KEYWORDS: single-layer reticulated shells, wind tunnel tests; limit load-carrying capacity; stability; the equivalent static method; the effective static loading distribution estimation method

1 INTRODUCTION

Reticulated shell structures are a kind of space-latticed system with the features of bar system structures and thin shells. For the structural design of single-layer reticulated shells, deformation and stability are the main problems, while the stress level in the elements is always not so high, usually just up to 2/3 of the permitted value of stress. During the structural analysis of such shells, geometrically non-linear behaviors are necessary to be considered. In addition, this structural system is very sensitive to the initial imperfections, such as the initial geometrical imperfection, etc. On the other hand, the difference between actual external loads that the structures will be subjected to at use stage and the estimated values at design stage, which can be taken as "load imperfections", also has important effects on the stability of shells. All the imperfections will possibly lead to an instability mode different from the predicted mode at design stage and corresponding to a different, usually lower limit load-carrying capacity (Gioncu, 1995 and Li, 1998). Such effects will increase seriously as the span of shells increases. At present, usually an equivalent static method based on the quasi-steady assumption is used with some empirical coefficients to reflect some dynamic characteristics. However, considering the random characteristics of fluctuating component, even with a larger empirical coefficient, it is hard to say that the structural design is safety enough. In this paper, based on wind tunnel tests, the effects of wind loading on single-layer reticulated shells, including on limit load-carrying capacity and stability, were investigated firstly. Then, a framework to estimate the effective wind loading distribution was introduced, and a new method from the stability concept point of view was presented to improve the efficiency of the effective static loading distribution estimation, as well as to give a conservatively estimation on the effect of wind loading in structural stability analysis. Finally, with comparative analyses, the advantage of the presented method was shown.

2 WIND TUNNEL TESTS WITH RIGID SHELL MODELS

2.1 *Wind tunnel and test conditions*

In order to know the characteristics of wind loading on shells, wind tunnel test on scaled models had been conducted in the boundary-layer wind tunnel (BLWT) of Wind Engineering Research Center, Tokyo Polytechnic University. It is an open-circuit low-speed boundary layer wind tunnel with 1.8m high, 2.2m wide, and about 19m long. With the spire-roughness technique, the expected wind profiles, Terrain type Ⅲ, which is according to the definitions in AIJ, 1996, was simulated, as shown Fig.1. In Fig.1, $E_r = U(z)/U_{10}^{\mathrm{II}}$, is the vertical distribution coefficient of wind speed in the flat uniformly rough (FUR) terrain, U_{10}^{II} is the wind speed at 10m high above Terrain type Ⅱ. As usual, the wind speed and the corresponding velocity pressure at the same height of the apex of models are taken as the reference wind speed and the reference pressure. Fig.2 gives the power spectral density (PSD) distribution of wind speed measured in the wind tunnel, which shows a good consistency with well-established Von Karman expression. The size of shell models and the distribution of measuring taps on their surface are shown in Fig.3.

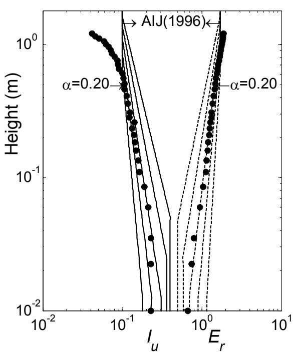


Figure 1.Wind profiles for Terrain type Ⅲ.

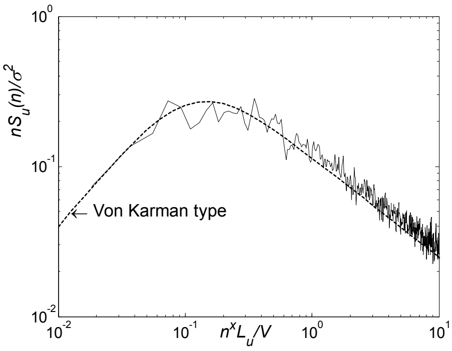
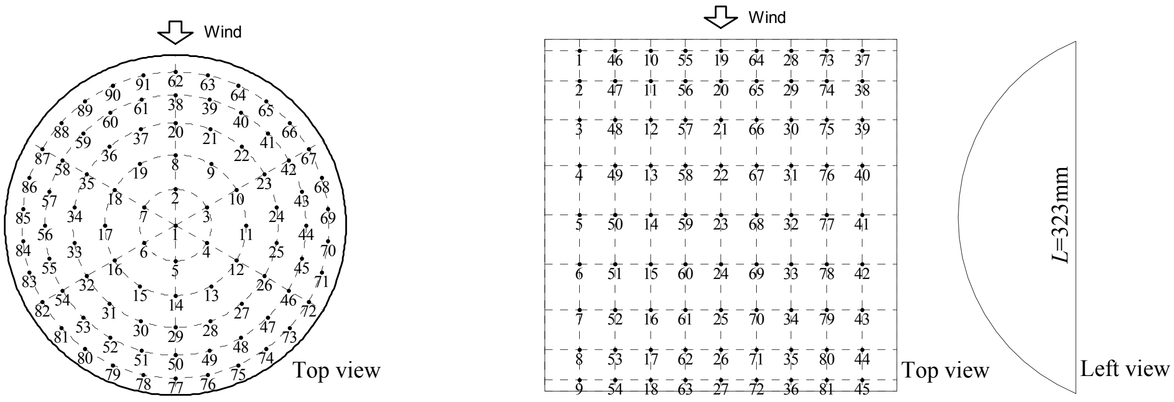


Figure 2. Longitudinal PSD.



(a) Spherical shell model (L=300mm, f/L=1/3) (b) Cylindrical shell model (B=L=323mm, f/L=1/3)

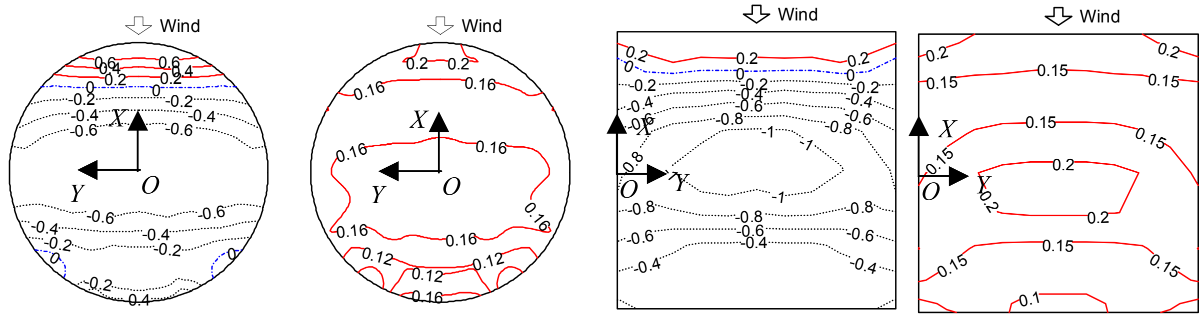
Figure 3. Shell models for wind tunnel tests and numbering of measuring taps.

2.2 *Experimental results*

Tab.1 gives the results of the mean and fluctuating wind pressure coefficient distribution measured simultaneously in the wind tunnel with a test wind speed of about 10 m/s. From Tab.1 we can find that, for both the spherical and the cylindrical shell, most area of their surfaces has suction except for a small part of their surfaces with positive pressure in the windward side.

Table 1. Distribution of the mean and the fluctuating wind pressure coefficients.

Spherical shell model		Cylindrical shell model	
Mean pressure	Fluctuating pressure	Mean pressure	Fluctuating pressure



3 EFFECTS OF FLUCTUATING WIND ON STRUCTURAL STABILITY AND LIMIT LOAD-CARRYING CAPACITY

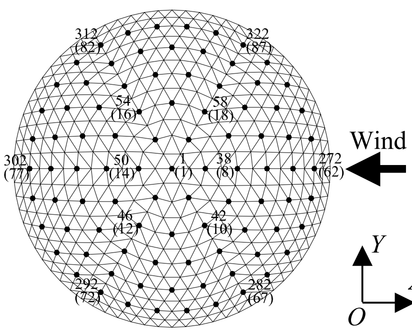
3.1 *Analysis model*

The sizes of models were decided according to the length scale, 1:400, in the wind tunnel tests.

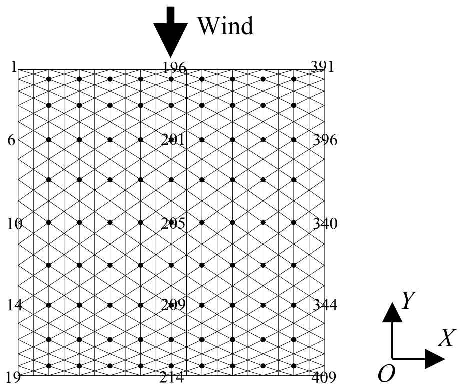
A Kewitt-type single-layer reticulated shell was chosen as the analysis model for spherical shell, as shown in Fig.4(a). The span L=120m, and the rise f = 40m. The sections of all elements in the model were assumed to be 200mm diameter tubes with 8mm thickness (ϕ200×8mm).

A translation type single-layer reticulated shell was chosen as the analysis model for cylindrical shell, as shown in Fig.4(b). L=129.2m and f=43.1m. The sections of all elements in the model were assumed to be 750mm diameter tubes with 24mm thickness (ϕ750×24mm).

All the joints on the bottom of shell models were assumed fixed for all degree-of-freedoms.



(a) A k6-12 single-layer reticulated spherical shell



(b) A translation type single-layer reticulated cylindrical shell

Figure 4. Analysis models

3.2 *Equivalent static analysis*

In an equivalent static wind-resistant analysis method for single-layer reticulated shells, the total static load vector $\{P\}$ in a load case including wind load can be expressed as follow:

$$\{P\} = C_D\{F_D\} + C_L\{F_L\} + C_W\{F_W\} \quad (1)$$

where $\{F_D\}$, $\{F_L\}$, and $\{F_W\}$ are the dead load vector, live load vector and equivalent static wind load vector, respectively; C_D, C_L and C_W is their corresponding load combination coefficients.

As for the wind load vector $\{F_W\}$, it can be divided into two parts: $\{\overline{F}_w\}$, resulted from the mean wind pressure on the surface of shells, and $\{\tilde{F}_W\}$, from the fluctuating wind pressure, as shown in following equation:

$$\{F_W\} = \{\overline{F}_w\} + \{\tilde{F}_W\} \tag{2}$$

As we all know, for stability tracing analysis, the incremental controlling equations for structural static nonlinear analysis have following expression:

$$[K_t]\{\Delta u\} = \{\Delta P\} + \{R\} \tag{3}$$

where $[K_t]$ is the current tangent stiffness matrix, $\{\Delta u\}$ is the displacement incremental vector, $\{\Delta P\}$ is the external load incremental vector, and $\{R\}$ is the residual force vector.

To resolve Eq.(3), the Arc-Length methods are widely used to trace the structural equilibrium paths. Generally, a proportional loading strategy is assumed, i.e. $\{\Delta P\} = \Delta\lambda\ \{P\}$, where $\Delta\lambda$ is the loading incremental parameter, $\{P\}$ is the external load reference vector. Then, the limit value of λ, λ_{cr}, can be used to represent the limit load-carrying capacity of structures.

In order to investigate the effects of fluctuating component of wind on the stability of single-layer reticulated shells, different loading combinations according to practical structural design processes were considered. Here a possible proportion of dead load, live load and wind load, 1:0.25:0.5, was assumed after the corresponding loading vectors were normalized by the maximum absolute values of their components, respectively. Following loading combination cases were analyzed: (a) $\{F_D\}+\{F_L\}$ (within full span); (b) $\{F_D\}+\{F_L\}$ (only within half span); (c) $\{F_D\}+\{F_W\}$ (with the same distribution of mean wind pressure); (d) $\{F_D\}+\{F_W\}$ (with the same distribution of maximum temporal wind pressure); (e) $\{F_D\}+\{F_W\}$ (with the same distribution of minimum temporal wind pressure); (f)~(j) $\{F_D\}+\{F_W\}$ (with the same distribution of temporal mean wind pressure at five random time points respectively within 1 second).

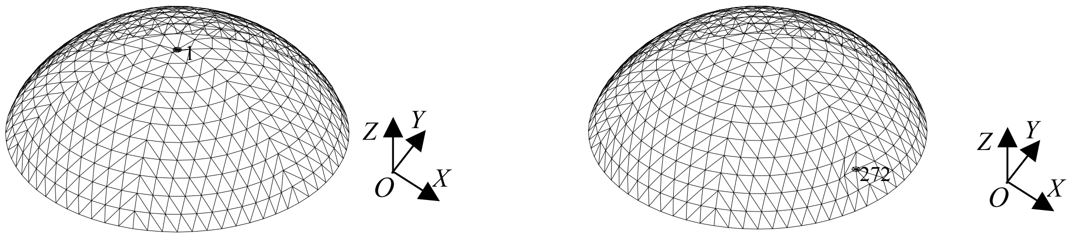
For the single-layer reticulated spherical shell model, Fig.5 gives two typical instability modes and their corresponding instability nodes under wind load. Fig.6 gives the loading-displacement paths in stability tracing analysis for all the analyzed cases. From Fig.6 we can find that, the effect of the amplitude of wind loading on single-layer reticulated shells can be easily understood, even with a linear extrapolation from the current loading level, i.e. λ=1. While the effects of the distribution of wind loading are really complex, different distributions may lead to different instability modes, and the difference among the limit load-carrying capacities corresponding to different instability modes are so big. Thus, we need to pay much attention to the distribution of wind loading on single-layer reticulated spherical shells, especially the effects of the fluctuating component of wind loading if using an equivalent static load method.

For the single-layer reticulated cylindrical shell model, Fig.7 gives two typical instability modes, i.e. a symmetrical mode and an anti-symmetrical mode. Fig.8 gives the loading-displacement paths in stability tracing analysis for all the cases. From Fig.8 we can find that, the effect of the wind loading distribution on the stability of the cylindrical shell model is obvious, although it is evidently smaller than that of the spherical shell model. The reason is that the anti-symmetrical mode is always the most disadvantageous instability mode that can be resulted from any an unsymmetrical loading distribution, such as a load combination case including wind load.

3.3 *Dynamic analysis*

Based on the theory of the finite element method, the nonlinear vibration equations of single-layer reticulated shell structures for wind-excited vibration can be described as follows:

$$[M]\{\ddot{U}(t)\} + [C]\{\dot{U}(t)\} + [K]\{U(t)\} = (\{F_D\} + \{F_L\}) + \{F_W(t)\} \tag{4}$$



(a) At Node 1 (b) At Node 272

Figure 5. Typical instability modes and their corresponding instability nodes for the spherical shell model.

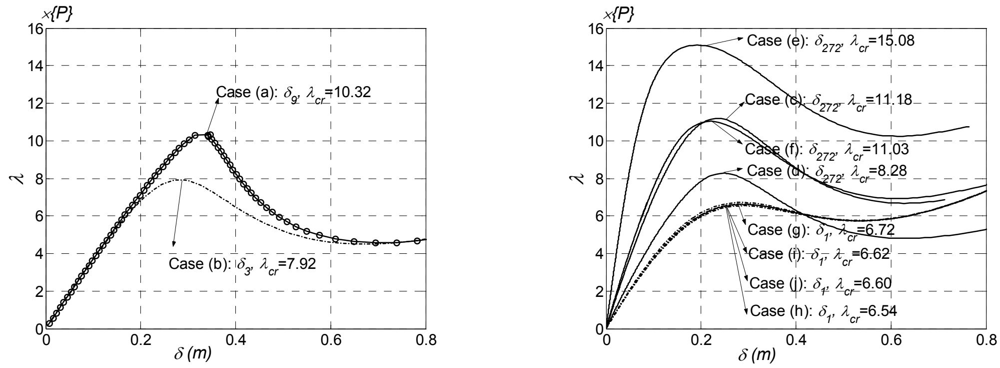
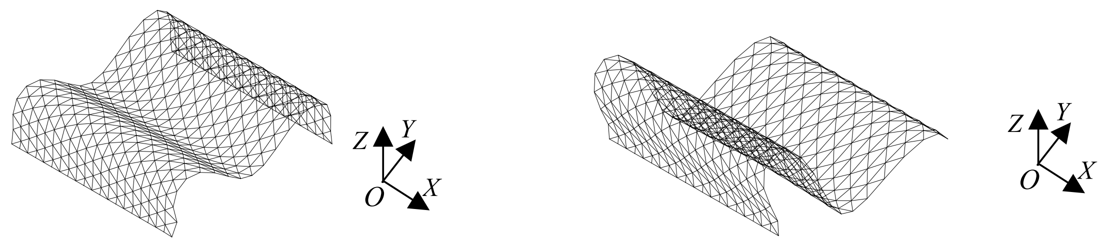


Figure 6. Loading-displacement paths in stability tracing analysis for the spherical shell model.



(a) The symmetrical mode (b) The anti-symmetrical mode

Figure 7.Typical instability modes for the cylindrical shell model.

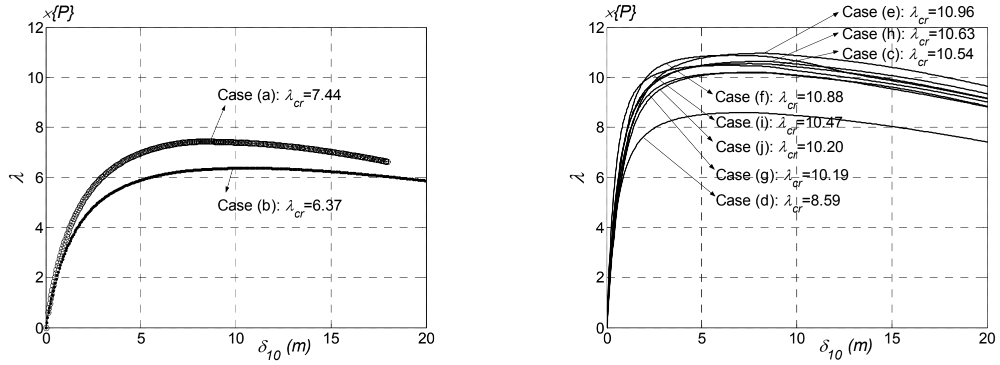


Figure 8. Loading-displacement paths in stability tracing analysis for the cylindrical shell model.

where, $[M]$ is the mass matrix; $[C]$ the damping matrix; $[K]$ is the nonlinear stiffness matrix of structures; $\{U(t)\}$, $\{\dot{U}(t)\}$ and $\{\ddot{U}(t)\}$ are the displacement, velocity and acceleration vectors, respectively; $\{F_W(t)\}$ is the time-history vector of the wind load.

With the wind pressure data measured simultaneously in wind tunnel tests, dynamic stability analyses for the spherical and cylindrical shell models were conducted using a dynamic nonlinear analysis method presented by Li and Tamura, 2002. With this method, dynamic stability can be checked conveniently during the nonlinear iterative analysis at each time step by the characteristics of the equivalent current tangent stiffness matrix as well as the maximum current deformation compared with the corresponding results from static instability analysis.

Compared with the results from above equivalent static analysis, the limit load-carrying capacities obtained from dynamic stability analysis are much smaller for both the single-layer reticulated spherical and the single-layer reticulated cylindrical shells, as shown in Tab.2.

4 THE EQUIVALENT WIND LOADING DISTRIBUTION ESTIMATION

4.1 T*he effective static loading distribution estimation method*

The equivalent static wind loading distribution can be estimated by the effective static loading distribution estimation to get the most unfavorable displacement at an expected node of shells.

The effective static wind loading distribution for a structure can be separately derived for three components as follows (Holmes, 2001):

$$\{F_i\} = \{\overline{F}_i\} + W_B\{F_{Bi}\} + W_R\{F_{Ri}\} \tag{5}$$

Where, $\overline{F}_i$, F_{Bi} and F_{Ri} are the mean, the background or sub-resonant and the resonant component, respectively, at node *i*. W_B and W_R are the weighting factors given by

$$|W_B| = \frac{g_B\sigma_{r,B}}{(g_B^2\sigma_{r,B}^2 + g_R^2\sigma_{r,R}^2)^{1/2}}, \quad |W_R| = \frac{g_R\sigma_{r,R}}{(g_B^2\sigma_{r,B}^2 + g_R^2\sigma_{r,R}^2)^{1/2}} \tag{6}$$

where g_B and g_R are the peak factors of background and resonant component; $\sigma_{r,B}$ and $\sigma_{r,R}$ are the standard deviations of background and resonant component of a response variable of interest, *r*.

The mean component of wind force at node *i* can be estimated as

$$\overline{F}_i = \overline{C}_{pi} q_h A_i = \overline{C}_{pi} \frac{1}{2} \rho_a \overline{V}_h^2 A_i \tag{7}$$

where $\overline{C}_{pi}$ is the mean wind pressure coefficient at node *i*, q_h is the reference pressure at the reference height. ρ_a is the density of air, $\overline{V_h}$ is the reference mean wind speed, and A_i is the tributary area at node *i*.

Load-Response Correlation (LRC) Method was presented by Kasperski and Niemann in 1992 to estimate the background or sub-resonant component of wind force as follows:

$$F_{Bi} = g_B \rho_{r,pi} \sigma_{pi} A_i = g_B \rho_{r,pi} C'_{pi} q_h A_i \tag{8}$$

where the peak factor of background component, g_B, normally lies in the range 2.5 to 5, $\rho_{r,pi}$ is the correlation coefficient between the response component and the wind pressure at node *i*, C'_{pi} is the fluctuating wind pressure coefficient at node *i*.

The resonant component of wind force can be estimated based on the superposition of the inertial forces corresponding to the first *M* vibration modes as follow:

$$\{F_{Ri}\} = \sum_{j}^{M} W_{Rj}[M]\{\phi_j\} \tag{9}$$

where W_{Rj} is the weighting factors corresponding to the *j*-th vibration mode, $\{\phi_j\}$ is the *j*-th vibration mode of structures based on a vibration mode analysis.

With the effective static loading distribution estimation method, the wind loading distribution can be estimated for the purpose to obtain a maximum or minimum value of a response variable of interest, *r*. However, there is still a problem, this is, how to decide a response variable that can be used as a reference to get the most unfavorable distribution. In this paper, A new method presented later can be used to determine a suitable response variable for above estimation as well as to give a conservative estimation of wind loading effects on the stability of shells.

4.2 *The most unfavorable estimation from the stability point of view*

Since the main questions in structural design of single-layer reticulated shells are stability and deformation problems, and the random characteristics of fluctuating wind load can also be taken as a loading imperfection, we can use a possible instability modes of shells as the most unfavorable distribution of fluctuating wind load to give a conservative estimation on the effects of fluctuating wind. Such method, named as the conformable imperfection mode method, is often used for sensitivity analysis of imperfections, and has been proved effective for such questions by many researchers (Li, 1998). In the paper, a simple method based on the conformable imperfection mode method, named "the most unfavorable estimation method", is presented.

Supposing during the stability tracing analysis under a load combination only including the mean wind force vector $\{\overline{F_W}\}$, $[K_t]$ becomes non-positive at the i+1-th incremental step, which means a limit or bifurcation point was occurred. Then, the calculation goes back to the initial state of this step, and an eigenvalue analysis of $[K_t]$ is carried out to obtain the current possible instability modes. With a chosen possible instability mode $\{v\}$, usually the first mode described by the first eigenvector, a most unfavorable distribution of wind load can be estimated as:

$$\{F_W\} = \{\overline{F}_W\} + \{\varepsilon_i\}\sigma_{F_W} = \{\overline{F}_W\} + \{\varepsilon_i\} g \rho_a \overline{V}_h \sigma_v A_i \quad (10)$$

where, $\{\varepsilon_i\}$ is the normalized vector of the product, $[K_t]\{v\}$. Here, $\{\varepsilon_i\}$ was taken as the most unfavorable distribution of fluctuating component of wind load. g is a peak factor with a range of 2.5-5.0. A_i is the tributary area at node *i*.

In this method, the amplitude of fluctuating component was considered with a uniform peak factor g as usual. Since the fluctuating component is really a random variable, the possible instability mode, usually the first mode, was used as the most unfavorable estimation of the distribution of fluctuating component of wind load. Therefore, we can obtain a conservative estimation of the effects of fluctuating component on the structural deformation and stability, as well as a suitable response variable, i.e. the displacement at the instability node obtained from the method, for using the effective static loading distribution estimation method.

4.3 *Comparative analyses*

In order to check the efficiency of the methods for estimating wind loading distribution on single-layer reticulated shells in stability analysis, load-carrying capacities and instability modes of the spherical and cylindrical shell models under the estimated loads by different methods were comparatively analyzed. In this paper, we assumed the design basic wind pressure w_0=0.5kN/m^2 corresponding to a design wind speed V_h=28.28m/s^2. If C_D, C_L and C_W are equal to 1 simply, the distributed dead load and live load will be 1.0kN/m^2 and 0.25kN/m^2, respectively, with an assumption of a possible proportion of dead load, live load and wind load as 1.0:0.25:0.5 as before. The results obtained from above different methods are listed in Tab.2. In Tab.2, The gust factor type method (Solari,1990) based on the loading code of China (GBJ99-87,1989) was used.

From Tab.2 we can find that, (1) The most unfavorable estimation method can obtained the lowest limit load-carrying capacity, and the most unfavorable instability mode as well; (2) The effective static loading distribution estimation are effective to get a acceptable result compared with the dynamic stability analysis if the determination of the reference response variable is suitable; (3) Since the gust factor type methods mean that the distribution of fluctuating component of wind is as the same as the distribution of mean component, it is not suitable to be used to estimate the equivalent static wind load for single-layer reticulated shells, sometimes a large gust factor will lead to more unsafely from the stability point of view. (4) For the cylindrical shell model, the effect of the wind loading distribution on structural stability is obviously smaller than that of the spherical shell model since the anti-symmetrical mode is always the most disadvantageous instability mode which resulted from any an unsymmetrical loading distribution, such as a load combination case including wind load.

Table 2 Limit load-carrying capacities and instability modes from different methods

Analysis methods	Spherical shell model		Cylindrical shell model	
	λ_{cr}	Mode	λ_{cr}	Mode
(a) The gust factor method	16.2	Fig.5(a)	15.9	Fig.6(b)
(b) The effective static loading distribution estimation method	12.6 (Node 1 in -*Z*)*	Fig.5(a)	10.9 (Node 205 in -*Z*)*	Fig.6(b)
	11.8 (Node 272 in -*X*)*	Fig.5(b)	10.7 (Node 201 in -*X*)*	Fig.6(b)
(c) The most unfavorable estimation method	6.50	Fig.5(b)	7.56	Fig.6(b)
(d) The dynamic stability analysis method	12.0	Fig.5(b)	10.5	Fig.6(b)

* The reference response variable (displacement) used in the effective static loading distribution estimation method

5. SUMMARY

In an equivalent static wind-resistant analysis for single-layer reticulated shells, estimation of the effective wind loading distribution should consider the effects of fluctuating component on structural stability. Combined with the most unfavorable estimation method presented by the authors in the paper, the effective static loading distribution estimation method can be used efficiently to estimate the effective wind loading distribution on such kinds of shell structures for stability analysis, which was proved by comparative analyses using the wind pressure data measured simultaneously from wind tunnel tests.

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